AQA

A-level
MATHEMATICS
7357/3

Paper 3

Mark scheme

June 2022

Version: 1.1 Final Mark Scheme

226A7357/3/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
ISW	Ignore subsequent working
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

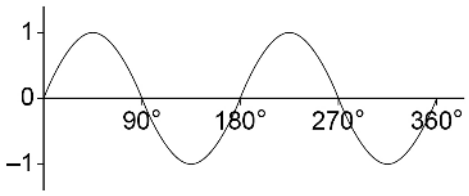
When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	$\lvert x\rvert < 4$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	3.1a	B1	$\int_0^5 (5x - x^2)\,\mathrm{d}x$
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	3.1a	B1	$x = -1$
	Question 3 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Integrates one x term correctly Accept unsimplified terms	1.1a	M1	$\frac{x^3}{3} + \frac{2x^{\frac{3}{2}}}{3} + c$
	Obtains correct answer ACF Must include + c Accept unsimplified terms Do not ISW	1.1b	A1	
	Question 4 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Sketches sine wave with correct orientation through the origin to at least one period Ignore any numbers on y-axis	1.1a	M1	
	Sketches $y = \sin 2x$ through the correct axes intersections Condone only slight difference in amplitudes Ignore any numbers on y-axis Ignore any sections of the graph outside of $0^\circ \leq x \leq 360^\circ$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Deduces correct values of A CAO	2.2a	B1	±1
	Subtotal		**1**	

	Question 5 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Substitutes t = 9.5 into $x = -2t^2$ or $2t^2$	3.4	M1	$t = 9.5 \Rightarrow x = -2 \times 9.5^2 = -180.5$ Length = 180.5 cm
	Obtains 180.5 Condone incorrect or missing units ISW	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(i)	Obtains 9 – 1.4t or –4t OE Ignore labels	1.1b	B1	$\frac{dy}{dt} = 9 - 1.4t$ $\frac{dx}{dt} = -4t$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $\frac{dy}{dx} = \frac{9 - 1.4t}{-4t}$
	Uses chain rule to obtain $\frac{dy}{dx}$ Condone sign error	3.1a	M1	
	Obtains a correct expression Do not ISW	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(ii)	Equates their $\frac{dy}{dx}$ or their $\frac{dy}{dt}$ or their numerator of their $\frac{dy}{dx}$ to 0 PI by correct t from correct $\frac{dy}{dx}$	3.1a	M1	$\frac{9-1.4t}{-4t}=0$ $t=\frac{45}{7}=6.43$ $y=9\times6.43-0.7\times(6.43)^2$ $=28.9$
	Obtains correct value for t ACF eg t = 6.4 or $\frac{9}{1.4}$ Must come from correct $\frac{dy}{dx}$	1.1b	A1	Width of surfboard = 58 cm $180.5 \div 3 = 60.2 \approx 58$ Hence the width is approximately one third of the length
	Substitutes their value for t into the model for y and obtains a value for y provided $0 < t < 9.5$	3.4	M1	
	Compares correct width and correct length and $\frac{1}{3}$ or 3 with a correct concluding statement OE CSO Allow 180 for length	3.2a	R1	
	Subtotal		**4**	

	Question 6 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Forms a correct expression for the gradient or sets up two correct simultaneous equations PI by $a = -0.7$ or $b = 1.5$ Ignore missing labels	1.1a	M1	$\frac{4.49-1.94}{3.46-1.76}=1.5$ $\log_{10} T-1.94=1.5(\log_{10} d-1.76)$ $\log_{10} T=-0.7+1.5\log_{10} d$
	Obtains $a = -0.7$ or $b = 1.5$ OE Ignore missing labels	1.1b	A1	
	Obtains $a = -0.7$ and $b = 1.5$ or seen in the logarithmic equation ISW	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(ii)	Uses one law of logarithm correctly Allow use of original equation without values for a and b If values are used, $a \neq 0$	3.3	M1	$\log_{10} T-\log_{10} d^{1.5}=-0.7$ $\log_{10}\left(\frac{T}{d^{1.5}}\right)=-0.7$
	Completes reasoned argument to obtain $T = Kd^n$ with $K = 10^{-0.7}$ or AWRT 0.2 and $n = 1.5$ ISW Must come from correct working	2.1	R1	$\frac{T}{d^{1.5}}=10^{-0.7}$ $T=10^{-0.7}\times d^{1.5}$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Forms an equation using their answer to **(a)(ii)** with K > 0 and n > 0 and $T = 60\,000$ Must only have unknown d in the equation	3.4	M1	$60\,000 = 0.2 \times d^{1.5}$ $d = 4488.5$ Average distance is approximately 4500 million kilometres
	Obtains AWRT 4500 million kilometres ACF with units For example • 4.5×10^9 **kilometres** • 4500×10^6 **kilometres** • 4.5×10^{12} **metres** • 4500×10^9 **metres**	3.2a	A1	
	Subtotal		**2**	

	Question 7 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Obtains $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{3\pi h^2}{12}$ or $\frac{\pi h^2}{4}$ OE Condone missing or incorrect labels	1.1b	B1	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{3\pi h^2}{12}=\frac{\pi h^2}{4}$ When $t = 3$ $V=\frac{\pi h^3}{12}=24$ $\Rightarrow h=\left(\frac{288}{\pi}\right)^{\frac{1}{3}}$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{4}\left(\frac{288}{\pi}\right)^{\frac{2}{3}}$ $=\pi^{\frac{1}{3}}\times\frac{1}{4}\times 82944^{\frac{1}{3}}$ $=\sqrt[3]{1296\pi}$ $=\sqrt[3]{216\times 6\pi}$ $=6\sqrt[3]{6\pi}$
	Obtains $v = 8 \times 3$ or 24 Can be embedded eg 288 or 96 × 3	3.1b	B1	
	Equates their 24 to $\frac{\pi h^3}{12}$ to obtain $h=\sqrt[3]{\frac{24\times 12}{\pi}}$ or $h^2=\left(\frac{288}{\pi}\right)^{\frac{2}{3}}$ Can be embedded Condone decimal values $h = 4.51$ $h^2 = 20.3$	1.1b	M1	
	Completes reasoned argument to show given result AG Must include $\frac{\mathrm{d}V}{\mathrm{d}h}$ with at least one intermediate step without 288 Must not include incorrect working in the manipulation	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	States any correct chain rule connecting $\frac{\mathrm{d}V}{\mathrm{d}t}, \frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}h}{\mathrm{d}t}$ PI by $\frac{8}{6\sqrt[3]{6\pi}}$ or correct answer or states that $h = \sqrt[3]{\frac{96t}{\pi}}$	3.1b	M1	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V}$ $\Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{8}{6\sqrt[3]{6\pi}}$ = 0.501 cm s^{-1}
	Substitutes $\frac{\mathrm{d}V}{\mathrm{d}t} = 8$ and $\frac{\mathrm{d}V}{\mathrm{d}h} = 6\sqrt[3]{6\pi}$ in their chain rule PI by $\frac{8}{6\sqrt[3]{6\pi}}$ or correct answer or substitutes t = 3 in their $\frac{\mathrm{d}h}{\mathrm{d}t} = (\frac{96}{\pi})^{\frac{1}{3}} \times \frac{t^{-2/3}}{3}$ ACF	1.1a	M1	
	Obtains correct $\frac{\mathrm{d}h}{\mathrm{d}t}$ AWRT 0.501 cm/s Must be at least 3sf with correct unit **cm/s** or **cm s^{-1}**	3.2a	A1	
	Subtotal		**3**	

	Question 8 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Begins argument with either of: • Factorises $-4b-2$ or $4b + 2$ correctly to 2b+1 as a factor • States $4b$ and 2 are both even or begins proof by contradiction by assuming a is odd therefore a^2 is odd	2.1	M1	$a^2 - 4b - 2 = 0$ $a^2 = 4b + 2$ $a^2 = 2(2b + 1)$ Hence a^2 must be even, which means that a must be even
	Completes reasoned argument by deducing that a^2 must be even or has a factor of 2, which means that a must be even or Completes reasoned argument by deducing that a^2 = 4b+2 which is even because $4b$ and 2 are both even hence a^2 is even which is a contradiction OE	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Uses $2p$ and obtains $(2p)^2$ PI by $4p^2$ Allow any letter for p except a and b	1.1a	M1	$a = 2p \Rightarrow (2p)^2 = 4p^2$ $4p^2 = 2(2b+1)$ $2p^2 = 2b+1$ Hence $2b+1$ must be even $2b+1$ is an odd number which is a contradiction
	Obtains either $4p^2 = 2(2b+1)$ or $4p^2 = 4b+2$ and followed by $2p^2 = 2b+1$ or $4p^2 = 2(2b+1)$ or $4p^2 = 4b+2$ and followed by $2 \times 2p^2 = 2(2b+1)$	3.1a	A1	
	Complete reasoned argument by deducing that $2b + 1$ is even hence contradiction as $2b+1$ is an odd number or Complete reasoned argument by deducing that $2b + 1$ is even hence contradiction as b cannot be an integer	2.2a	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Deduces that there are no solutions to $a^2 - 4b - 2 = 0$ where a **and** b are integers	2.2a	R1	There are no solutions to $a^2 - 4b - 2 = 0$ where a and b are integers
	Subtotal		**1**	

	Question 9 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	States -2.5 OE	2.2a	B1	-2.5
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Explains that many-to-one function is when distinct values of x give the same value for y	2.4	E1	Many-to-one function is when two or more x values give the same y value. This graph is many-to-one because you can draw a horizontal line and it will cross the graph twice.
	Uses the shape of the graph to justify their answer or gives an example of two x values eg $\mathrm{f}(0)=\mathrm{f}(4)$ or states turning or minimum or maximum points indicate many-to-one	2.4	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(i)	Equates x and $\dfrac{x^2+10}{2x+5}$	3.1a	M1	$x=\dfrac{x^2+10}{2x+5}$ $x(2x+5)=x^2+10$ $2x^2+5x=x^2+10$ $x^2+5x-10=0$
	Rearranges with at least one intermediate step to obtain quadratic equation AG Condone $0 = x^2 + 5x - 10$	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(ii)	Obtains $\dfrac{-5\pm\sqrt{65}}{2}$ Ignore any labels ISW	1.1b	B1	$x=\dfrac{-5\pm\sqrt{65}}{2}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)	Uses quotient rule to obtain an expression in the form of $\frac{Ax(2x+5)+B(x^2+10)}{(2x+5)^2}$ or uses product rule to obtain an expression in the form of $Cx(2x+5)^{-1}+D(x^2+10)(2x+5)^{-2}$ or uses implicit differentiation to obtain an equation of the form $Ax\frac{dy}{dx}+By+C\frac{dy}{dx}=Dx$ A, B, C and D can be any values but not 0 Condone missing brackets	3.1a	M1	$f'(x)=\frac{2x(2x+5)-2(x^2+10)}{(2x+5)^2}$ $=\frac{2x^2+10x-20}{(2x+5)^2}$ $f'(x)=0 \Leftrightarrow 2x^2+10x-20=0$ $x^2+5x-10=0$ This is the same equation solved in part c(i) so P and Q must be stationary points.
	Obtains fully correct $f'(x)$ or obtains $2x\frac{dy}{dx}+2y+5\frac{dy}{dx}=2x$ ACF May be unsimplified	1.1b	A1	
	Equates their $f'(x)$ or their numerator of $f'(x)$ to 0 or sets $\frac{dy}{dx}=0$	1.1a	M1	
	Rearranges to obtain $x^2 + 5x - 10 = 0$ or $2x^2 + 10x - 20 = 0$ and links it to the equation in part **c(i)** or their answer to **c(ii)** or solves their quadratic $f'(x)=0$ correctly or deduces $y = x$ and substitutes to get $x=\frac{x^2+10}{2x+5}$ then rearranges to get $x^2 + 5x - 10 = 0$	1.1a	M1	

	Completes a reasoned argument by using $x = \frac{-5 \pm \sqrt{65}}{2}$ to conclude that P and Q are stationary points CSO Must have brackets correct throughout	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(e)	Deduces critical regions from their answer to **c(ii)** condone strict inequalities or poor notation or decimal values	2.2a	M1	$x \leq \frac{-5-\sqrt{65}}{2}$ and $x \geq \frac{-5+\sqrt{65}}{2}$ $\left\{x : x \leq \frac{-5-\sqrt{65}}{2}\right\} \cup \left\{x : x \geq \frac{-5+\sqrt{65}}{2}\right\}$
	Writes correct range in correct set notation eg $\left(-\infty, \frac{-5-\sqrt{65}}{2}\right] \cup \left[\frac{-5+\sqrt{65}}{2}, \infty\right)$ Accept other letters for x or using $f(x)$ provided consistent throughout set Follow through their answer to **c(ii)**	2.5	A1F	
	Subtotal		**2**	

	Question 10 Total		**13**	

Q	Marking instructions	AO	Marks	Typical solution
11	Circles correct answer	1.1b	B1	0.59
	Question 11 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12	Circles correct answer	2.2a	B1	Positively skewed
	Question 12 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13	Makes a valid comment about the sample of vehicles in LDS For example • not all (UK/England) regions included • not all types of vehicles included • some missing CO_2 data	2.2b	E1	The Large Data Set is only a sample of vehicles in the UK. The Large Data Set only has data for 2002 and 2016
	Makes a valid comment about the data years in LDS For example • data shown only for two years but nothing in between	2.2b	E1	
	Question 13 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	States one of the following assumptions in context • probability of complaint call is constant • calls or complaint calls occur independently of each other or have no effect on each other • only two outcomes of complaint or non-complaint calls Condone 'complaint' for 'complaint call' Do not allow probability being independent Do not allow fixed number of calls	3.5b	B1	The probability of getting a complaint call is fixed Calls occur independently of each other
	States a second assumption in context	3.5b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)(i)	Calculates the correct probability ACF AWFW [0.0068, 0.007]	1.1b	B1	0.00684
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)(ii)	Finds $P(X \leq 4) or P(X \leq 3)$ PI by 0.2375 or 0.107 Allow figures to 2sf	1.1a	M1	$P(X \leq 3)$ =0.107
	Obtains correct probability ACF AWFW [0.107, 0.11]	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)(iii)	Finds their $P(X \leq 9)$ or $P(X \leq 10)$ or $P(X \geq 10)$ or $P(X > 10)$ PI by 0.952 or 0.983 or 0.0172 or correct answer Allow figures to 2sf	1.1a	M1	$P(X \geq 10) = 1 - P(X \leq 9)$ $= 1 - 0.952$ $= 0.048$
	Obtains correct probability ACF AWFW [0.0479, 0.048]	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)	Uses $np(1-p)$	1.1a	M1	
	Forms a correct equation in p using 10 and 1.5 ACF	3.1a	A1	$10p(1-p) = 1.5^2$ $-10p^2 + 10p - 2.25 = 0$
	Obtains values for p ACF ISW Allow AWFW [0.34, 0.342] and [0.658, 0.66] for p or $p = \frac{10 \pm \sqrt{10}}{20}$	1.1b	A1	$p =$ 0.34 and 0.66
	Subtotal		**3**	

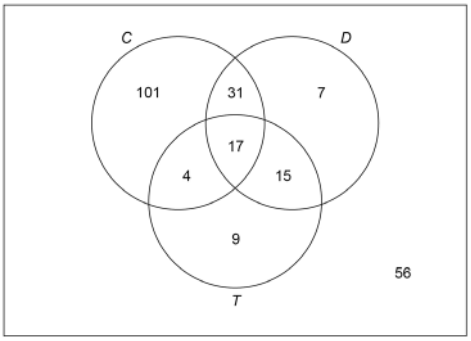
	Question 14 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	States the correct population Allow 'people' or 'names' or 'members' for adults Must see electoral register	3.3	B1	Adults on the electoral register
	Subtotal		**1**	

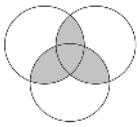
Q	Marking instructions	AO	Marks	Typical solution
15(b)(i)	Recalls correct name for sampling method	1.2	B1	Systematic sampling
	Subtotal		**1**	

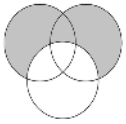
Q	Marking instructions	AO	Marks	Typical solution
15(b)(ii)	States one advantage that sampling method For example • easy to carry out or quick • no bias or it is fair • reduce chance of cluster Do not accept random Do not ignore incorrect statement	3.5b	E1	Cheap to collect
	Subtotal		**1**	

	Question 15 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	Writes at least two of 4,15, 17 or 31 in the correct place	1.1a	M1	
	Obtains either 7 or 9 in the correct place	1.1b	A1	
	Completes the Venn diagram fully correctly including 56	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(i)	Obtains the correct probability AWFW [0.42, 0.421] Ignore subsequent incorrect simplification once correct fraction or decimals obtained	1.1b	B1	$\frac{101}{240}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(ii)	Adds their 4 + 15 + 17 + 31  PI by correct answer OE The total is not required at this stage	1.1a	M1	4 + 15 + 17 + 31 = 67 $\frac{67}{240}$
	Obtains the correct probability AWFW [0.279, 0.28] Ignore subsequent incorrect simplification once correct fraction or decimals obtained	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(iii)	Adds their 101 + 31 + 7  PI by correct answer OE The total is not required at this stage	1.1a	M1	101 + 31 + 7 = 139 $\frac{139}{195}$
	Obtains the correct probability AWFW [0.71, 0.713] Ignore subsequent incorrect simplification once correct fraction or decimals obtained	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
16(c)	Finds correct $P(C) \times P(T)$ or uses correct conditional probability eg $P(T\|C) = \frac{21}{153}$ OE All figures must be correct	3.1b	M1	$P(C) \times P(T) = \frac{153}{240} \times \frac{45}{240} = \frac{153}{1280}$ $= 0.12$ $P(C \cap T) = \frac{21}{240}$ $= 0.0875$ $0.12 \neq 0.0875$ Hence events C and T are not independent
	Compares $P(C \cap T) = \frac{21}{240}$ with $\frac{153}{1280}$ and concludes owning a cat and owning a tortoise are not independent eg $\frac{21}{240} \neq \frac{153}{1280}$ so not independent or compares conditional probability eg $\frac{21}{153} \neq \frac{45}{240}$ so not independent All figures must be correct	2.2a	R1	
	Subtotal		**2**	

	Question 16 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
17	States both hypotheses correctly for one-tailed test	2.5	B1	X = working hours per week $H_0: \mu = 34$ $H_1: \mu > 34$
	States or uses correct model PI by mean 34 and variance $\frac{4.5^2}{30}$ or standard deviation $\frac{4.5}{\sqrt{30}}$ OE or by correct probability AWFW [0.0037, 0.004] or test statistic $\pm\frac{36.2-34}{4.5\div\sqrt{30}}$ or test statistic value AWFW ±[2.67, 2.68] or critical region > AWFW [35.6, 35.632] condone ≥ or critical value $\pm$ [35.6, 35.632]	1.1a	M1	$\bar{X} \sim N(34, \frac{4.5^2}{30})$ $P(\bar{X} > 36.2) = 0.004$ 0.004 < 0.025 Reject H_0 There is sufficient evidence to suggest that the mean working hours have increased.
	Obtains AWFW [0.0037, 0.004] or obtains the correct value of the test statistic AWFW [2.67, 2.68] or obtains the correct critical region > AWFW [35.6, 35.632] condone ≥ condone incorrect missing or incorrect labels	1.1b	A1	
	Compares their [0.0037, 0.004] with 0.025 or compares their [2.67, 2.68] with 1.96 allow positive values up to 4 for 1.96 or compares their region > [35.6, 35.632] with 36.2 condone ≥	3.5a	M1	

	Infers H_0 or null hypothesis rejected Condone H_1 or alternative hypothesis accepted All figures must be correct	2.2b	A1	
	Concludes correctly in context that there is **sufficient evidence** to **suggest** that the **mean** working **hours** have **increased** To be awarded R1, marks M1A1M1A1 must be scored as the minimum	3.2a	R1	
	Question 17 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
18(a)	Calculates $1.78 \pm 2 \times 0.23$ or $1.78 \pm 1.96 \times 0.23$ or calculates $P(1.33 < x < 2.22)$ PI by 0.9469 or 0.947 or calculates $\frac{1.33 - 1.78}{0.23}$ and $\frac{2.22 - 1.78}{0.23}$	3.1b	M1	$1.78 - 2 \times 0.23 = 1.32$ $1.78 + 2 \times 0.23 = 2.24$ $1.32 \approx 1.33$ $2.24 \approx 2.22$ Height is continuous data and 95% of heights lies within two standard deviations of the mean so normal may be a suitable model.
	Obtains 1.32 and 2.24 and states they are approximately 1.33 and 2.22 or obtains 1.33 and 2.23 and states they are approximately 1.33 and 2.22 or obtains 0.9469 or 0.947 and states is approximately 0.95 or obtains -1.96 and 1.91 and states are approximately -2 and 2	2.4	A1	
	Infers that the normal distribution may be suitable because **height** is **continuous** data and **95%** of heights lies within two standard deviations of the mean or Infers that the normal distribution may be suitable because **height** is **continuous** data and **94.69%** or **94.7%** of heights lies between 1.33 and 2.22	2.2b	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)(i)	States 0	1.2	B1	0
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)(ii)	Calculates the correct probability AWFW [0.335, 0.34]	1.1b	B1	0.335
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)(iii)	Finds the value of their answer to **(b)(ii)** squared Their answer must be correct to at least 2sf	3.1b	B1F	$0.335^2 = 0.112$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(c)	Obtains 1.73 CAO Ignore missing or incorrect units	1.1b	B1	Mean = 1.73 Standard deviation $= \sqrt{\frac{2.81}{40}}$ $= 0.265$
	Uses the correct formula for standard deviation eg $s = \sqrt{\frac{2.81}{39}}$ Do not allow variance $= \sqrt{\frac{2.81}{40}}$	1.1a	M1	
	Obtains the correct standard deviation AWFW [0.265, 0.27] Allow if not labelled but if labelled, must be correct Ignore missing or incorrect units	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
18(d)	Uses their mean and 1.78 to compare heights Comparison must include on average. Follow through their answer to part **(c)** Do not allow 'general' Allow statement that they're about the same on average	2.2b	E1F	Summer athletes are taller on average than Winter athletes. Summer athletes' heights are less varied than the heights of Winter athletes.
	Uses their standard deviation and 0.23 to compare heights Comparison must include 'varies', 'spread' 'disperse' 'more variation' or 'consistent' Follow through their answer to part **(c)** Allow statement that they are about the same Do not allow comparison that includes 'range' or 'variety'	2.2b	E1F	
	Subtotal		**2**	

	Question 18 Total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
19	States both hypotheses correctly for one-tailed test 0.42 OE	2.5	B1	X = number of customers with an internet banking account H_0: p = 0.42 H_1: p > 0.42 Under null hypothesis $X \sim$B(35,0.42) $P(X \geq 18) = 0.169$ 0.169 > 0.10 Do not reject H_0 There is insufficient evidence to suggest an increase in the proportion of bank customers registered for an internet banking account.
	States or uses correct model PI by calculation of one of the probabilities below $P(X \leq 16) = [0.73, 0.733]$ $P(X \leq 17) = [0.83, 0.832]$ $P(X \leq 18) = [0.90, 0.903]$ $P(X \geq 18) = [0.168, 0.17]$ $P(X \geq 19) = [0.097, 0.10]$ or critical value of 19 or region ≥ 19	3.3	M1	
	Obtains $P(X \geq 18) = [0.168, 0.17]$ or Obtains a critical value 19 or critical region ≥19	1.1b	A1	
	Evaluates binomial model by comparing their $P(X \geq 18)$ with 0.10 or Evaluates binomial model by comparing their critical region with 18	3.5a	M1	
	Infers H_0 or null hypothesis not rejected Condone H_0 accepted or H_1 or alternative hypothesis rejected All figures must be correct	2.2b	A1	
	Concludes correctly in context that there is **insufficient evidence** to **suggest** an **increase** in the **proportion** of bank **customers** registered for an internet banking account To be awarded R1, marks M1A1M1A1 must be scored as the minimum	3.2a	R1	
	Question 19 Total		**6**	

	Question Paper Total		**100**	